Enhanced modelling of the non-gravitational forces acting on LAGEOS

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Abstract

LAGEOS-I and LAGEOS-II orbit Earth since 1976 and 1992 respectively. With 426 Corner Cube Reflectors (CCRs) embedded in a spherical surface and a very low area-to-mass ratio, the LAGEOS satellites are among the best tools for global space geodetic research. By means of SLR observations, geophysical phenomena such as variations of the Terrestrial Reference Frame (TRF) origin w.r.t. the geocenter, global scale, low-degree gravity field terms, Earth Orientation Parameters (EOPs) and plate tectonic motions can be accurately measured, their accuracy directly dependent on that of the ground laser instrumentation and the accuracy of the orbit determination.

Intensive orbital analyses yielded a decrease in the semi-major axis of the orbit of LAGEOS-I, at a rate of 1.3 mm/d, shortly after launch; a similar decay has been observed for LAGEOS-II. Various physical processes (or a combination of them) have been proposed as possible causes for this acceleration: radiation pressure from celestial bodies (Earth and Sun) mismodeling, thermal thrust (re-radiation from the satellite itself), together with eclipse dependencies of the (re-)radiation, and ionospheric drag (neutral and charged particles). This decay can be modeled by an empirical along-track acceleration with a mean value of about -3.4 pm/s^2. The modeling efforts done so far have given a partially successful explanation of the non-gravitational perturbations acting on LAGEOS. However, a clear signal is still present in the calculations, due to a lack of precise modeling of the (unique) physical truth.

This study has concentrated on an accurate modeling of the major factors which could be responsible of the unexplained signal: the geometrical and optical properties, the rotational dynamics of the spacecraft, and poorly modeled forces. Accurate results have been obtained for the rotational dynamics thus eliminating one of the largest uncertainties still present. In parallel, finite element modeling has permitted a detailed characterization of the various elements of the spacecraft, together with an accurate description of their (time-dependent) geometry w.r.t. radiation sources. This has yielded a numerical answer for the thermal accelerations for all possible spinning regimes. Uncertainties in some physical parameters have been dealt within a sensitivity analysis.

Introduction

Although the technique of Satellite Laser Ranging (SLR) dates back more than 40 years [Marshall *et al.*, 1995], it is still one of the main techniques to be used for studying certain elements of System Earth. In particular, global aspects of the terrestrial reference frame, such as origin and scale, are uniquely determined by this technique by virtue of its direct and unambiguous method of observation: the travel time measurements of a pulse of light from a ground station to a satellite and back are typically measurable with high precision, and the various elements that play a role in converting these 2-way travel times into a 1-way range observation (*e.g.* satellite signature, atmospheric refraction, station delay, etcetera) can be modeled with an accuracy of various mm typically [Otsubo and Appleby, 2005]. To arrive at the best possible solutions for such global parameters, it is mandatory to model the orbit of the satellites as accurately as possible. Typically, the cannonball satellites LAGEOS-1 and LAGEOS-2 (launched in May 1976 and October 1992, respectively) are used for this purpose by virtue of their attractive area-to-mass ratio, making them relatively insensitive to (intrinsically complex to model) surface forces.

In spite of the attractive design of these spacecraft, high-precision orbit determination currently necessitates the estimation of so-called empirical accelerations (typically, in various directions w.r.t. an orbit-referenced frame and with different character – constant or sinusoidal with orbital period). This is a clear indication of the limitations of current analysis models to represent “physical truth” correctly. An illustration of this is given in Figure 1, which shows the residuals of the constant along-track acceleration as observed/estimated for the satellite LAGEOS-1, *i.e.* bi-weekly solutions of such a parameter after subtraction of best known physical mechanisms to explain the acceleration (in reality, the accelerations show a mean value of -3.4 pm/s^2, which can be addressed to a variety of surface forces). The plot clearly illustrates that there is a signal in the residuals at the level of several pm/s^2, which needs a physical explanation in order to advance the contributions of LAGEOS–type missions to geophysical studies further. Candidates for the residuals shown here are (1) thermal radiation exerted by the satellite itself, (2) direct radiation forces, (3) charged and neutral particle drag, and others; of course shortcomings in the modeling of any of these individually, and/or a combination of effects can play a role here. This paper will focus on the so-called thermal forces: minute forces that are introduced by the emission of thermal energy by surface elements of a satellite.

First, a model for the rotational behavior of the satellites will be presented. Previous investigations by other authors show that a proper understanding and description of this aspect is crucial for a good modeling of the thermal behavior. The thermal behavior of the satellites will be the next topic of discussion, and a multi-node model of each satellite will be developed and used to simulate actual temperatures. Then, the temperature distribution will be used to compute contributions to thermal forces as exerted by individual surface elements, resulting in a total acceleration. This acceleration will be used in a first-order assessment of its orbital effect. The paper will end with conclusions and recommendations.

Rotational dynamics

Compared to the orbital motion of the spacecraft, the rotational dynamics of LAGEOS-1 and -2 can be considered as a neglected element of the mission: observations of the attitude and spin rate are few, and models of the rotational behavior are hardly available. One of the reasons for this is the absence of any need for such information: the rotation dynamics plays a subtle role in the orbital behavior of the vehicles, which only come into play when the requirements on orbital accuracies arrive at the level of a single cm and below. A recently developed description of the spin behavior of the LAGEOS pair is given in [Andrés *et al.*, 2004].

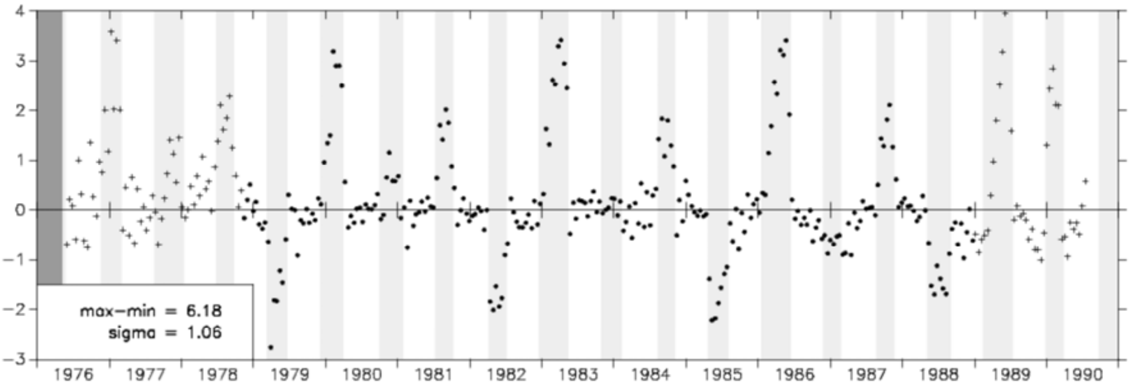

Figure 1. *Residuals of the along-track accelerations as observed for LAGEOS-1, for the time period 1976-1990. Grey areas indicate the periods when the satellite experienced an umbra while orbiting the Earth [Scharroo et al., 1991].*

The LAGEOS Spin Axis Model (LOSSAM) that is developed in this reference is based on a straightforward integration of Euler's equation:

$$\frac{d\vec{L}}{dt} = \vec{M}_{magn} + \vec{M}_{grav} + \vec{M}_{offset} + \vec{M}_{reflec}$$

Here, the external torques represent the influence of the Earth's magnetic field, gravity, a possible difference between the center-of-pressure w.r.t. the center-of-mass, and a possible difference in effective reflectivity between the northern and southern hemisphere of the satellites, respectively. LOSSAM has been obtained after confrontation of the theoretical model as described by the previous equation with independent observations on spin-axis orientation and spin rate taken by a variety of stations and institutes: (*i*) University of Maryland, USA, (*ii*) the laser station in Herstmonceux, UK (owned by the Natural Environment Research Council, NERC), (*iii*) the laser station in Matera, Italy (owned by the Agenzia Spaziale de Italia, ASI) and (*iv*) Lincoln Laboratory [Sullivan, 1980]. Figures 2 and 3 show the behavior of the spin axis orientation of LAGEOS-1 and -2 according to LOSSAM, respectively (spin rate results are withheld here). The plots also show the independent observations that were used in the derivation of the model, and the level of fit. Clearly visible is that LAGEOS-1 is in a different rotational regime currently than LAGEOS-2: the spin-axis orientation of the former satellite follows a more irregular pattern, which is due to a slowing down from a rotational period of 10.5 s at launch (1976) to about 6000 s now (Figure 2). LAGEOS-2 is still spinning with a period of about 360 s currently. Also visible is the fact that the set of observations on the spin axis that is available for LAGEOS-1 is quite restricted: the last ones were taken at the end of 1996, and effectively one cannot do but make predictions of the current behavior of the satellite; the absence of recent observations is directly related to the fact that the rotation of LAGEOS-1 has almost come to a standstill, which makes it extremely difficult to actually apply currently practiced observation techniques on spin axis orientation and rotation rate. For LAGEOS-2, the situation is much better (cf. Figure 3). The reader is referred to [Andrés *et al.*, 2004] for more details.

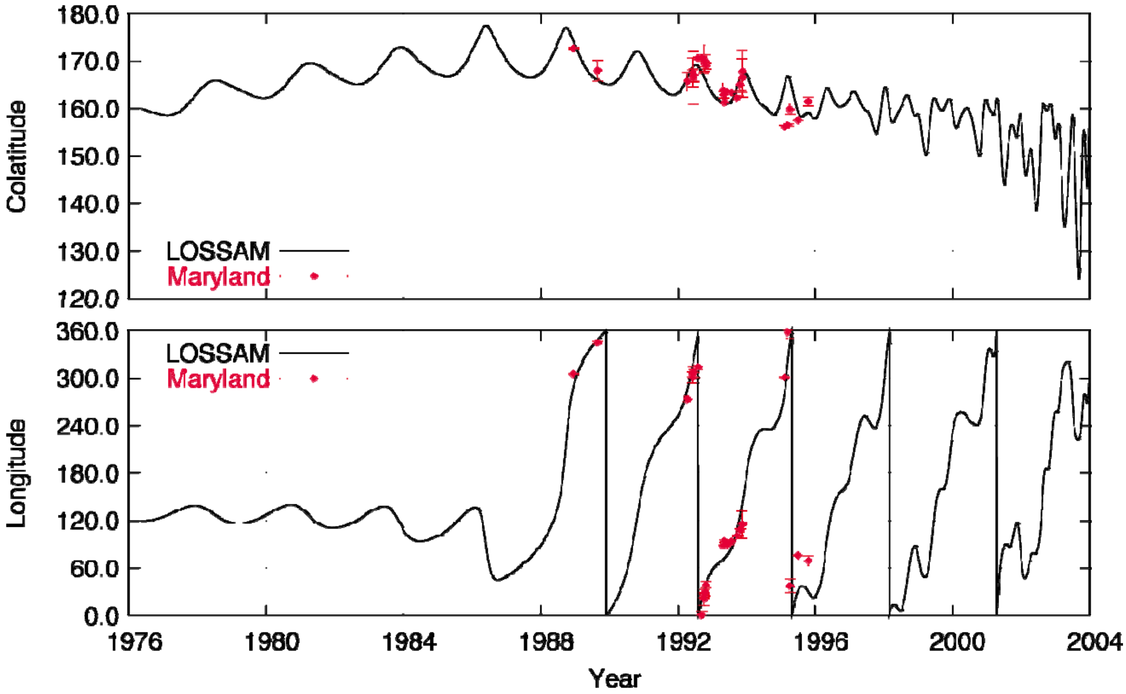

***Figure 2.** The LOSSAM spin orientation behavior of LAGEOS-1 as a function of time, as described by the longitude and co-latitude w.r.t. the J2000 reference frame. The red symbols represent the independent observations that were used to derive this model [Andrés et al., 2004].*

Thermal model

Thermal forces, *i.e.* forces that are generated somehow by either direct or reflected radiation, are known to play an important role in the explanation of the observed decay of the semi-major axis of the LAGEOS pair and, directly related to this, of the solutions for the empirical accelerations; many studies have been done into the effects of direct solar radiation (Yarkovsky effect), albedo radiation, earth infrared radiation, the effect of eclipses (Yarkovsky-Schach effect), etcetera (*e.g.* [Rubincam, 1982], [Anselmo *et al.*, 1983], [Barlier *et al.*, 1986], [Rubincam, 1987a], [Rubincam, 1987b], [Afonso *et al.*, 1989], [Rubincam, 1990], [Martin and Rubincam, 1996], [Slabinski, 1997] and [Vokrouhlický and Métris, 2004]). However, none of these investigations has led to a full description and complete understanding of the actual phenomena that influence the orbital behavior of the LAGEOS satellites; if only because simplifications had to be made in order to arrive at first-order estimates of the effects. Clearly, in view of the slow rotation of LAGEOS-1 and a similar trend for LAGEOS-2, the necessity for a more detailed modeling of the satellite and its interaction with various elements in its environment has arisen. As mentioned in the introduction, this paper addresses one of those elements: the thermal interaction with the various radiation sources, and the resulting accelerations. A detailed discussion of procedures, models and results is given in [Andrés *et al.*, 2006].

To model the interaction in detail, making allowance for potential differences in its reaction to various sources of energy, the satellite model needs to be split up into a number of different components. In recognition of the various mechanisms that are effectively responsible for heat transfer (*i.e.* radiation and conduction; any other can be shown to be insignificant [Andrés *et al.*, 2006]) and the differences in thermal and mechanical properties of the various construction elements, a finite-element model of each LAGEOS satellite has been created, with 2133 elements in total: the inner core

(core and stud), two hemispheres, and 426 retroreflector assemblies each consisting of 5 elements: a retainer ring, an upper ring, a corner-cube reflector, a set of ring posts, and a lower ring.

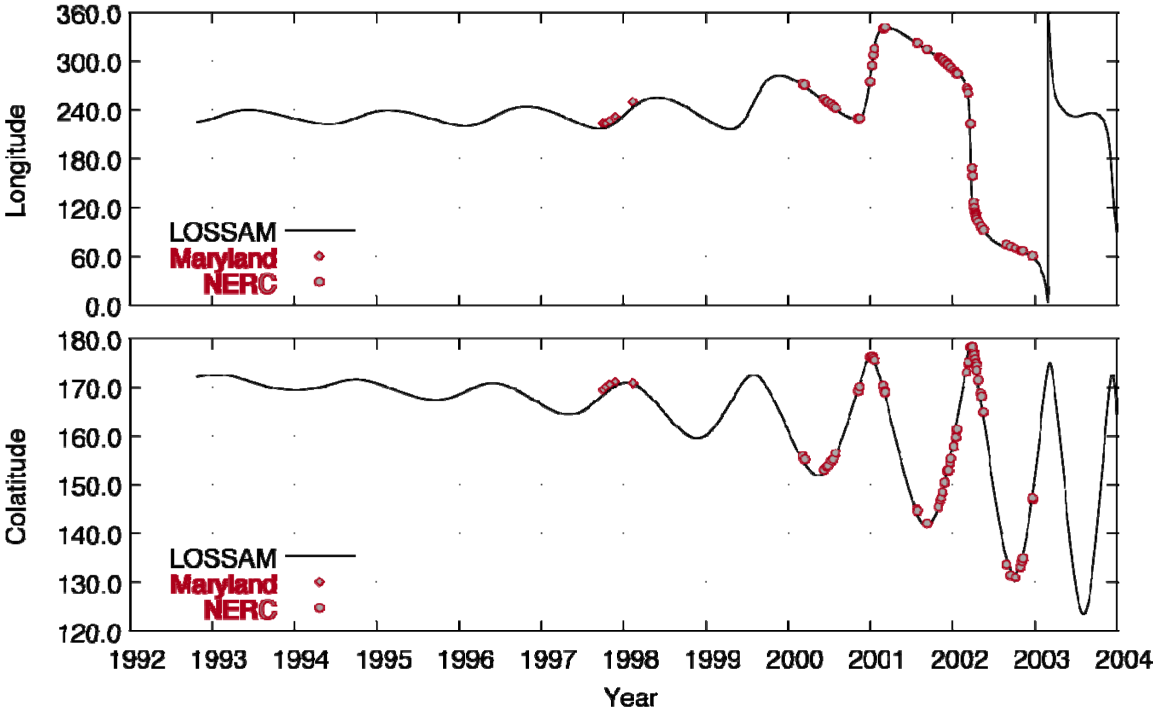

Figure 3. *The LOSSAM spin orientation behavior of LAGEOS-2 as a function of time, as described by the longitude and co-latitude w.r.t. the J2000 reference frame. The red symbols represent the independent observations that were used to derive this model [Andrés et al., 2004].*

For each LAGEOS element *i*, the following (abstract) heat equation can be written:

$$H_i \frac{\partial T_i}{\partial t} = Q_{in} - Q_{out}$$

For more details, see [Andrés *et al.*, 2006]. In combination with cm-level accurate solutions for the orbital motion of the satellites (obtained with GEODYN [Pavlis *et al.*, 1998], the positions of sources of radiative energy (Sun, Earth), models for these radiative flows, models for the thermal and mechanical properties of the spacecraft components, and the LOSSAM model for the rotational behavior of the spacecraft [Andrés *et al.*, 2004] this equation can be integrated over time for each element to yield the thermal behavior of each individual element. This has been done for both satellites from the date of launch onwards, with a step-size of 60 s, and taking care that allowance is made for aspects like shadowing, aliasing (when the rotational period and the integration step size are integer multiples) and rotationally averaged radiation input. An illustration of the result is given in Figure 4, which shows the temperature distribution of the various elements of LAGEOS-1 and -2 for the (arbitrary) epoch January 1, 2002, respectively. The plots clearly show the different temperatures of the Germanium reflectors (3 out of 4 are visible in each plot; the thermal absorption and emission coefficients are very different from the quantities for the 422 Silicium reflectors), and, in a similar fashion, the different temperatures for the retainer rings. In the case of LAGEOS-1 (Figure 4, left), the Sun is more-or-less located over the satellites equator, resulting in a similar temperature for the northern and the southern hemispheres. In the case of LAGEOS-2 (Figure 4, right), the Sun is at an apparent latitude of about 45°, with a higher temperature for the northern hemisphere as a consequence.

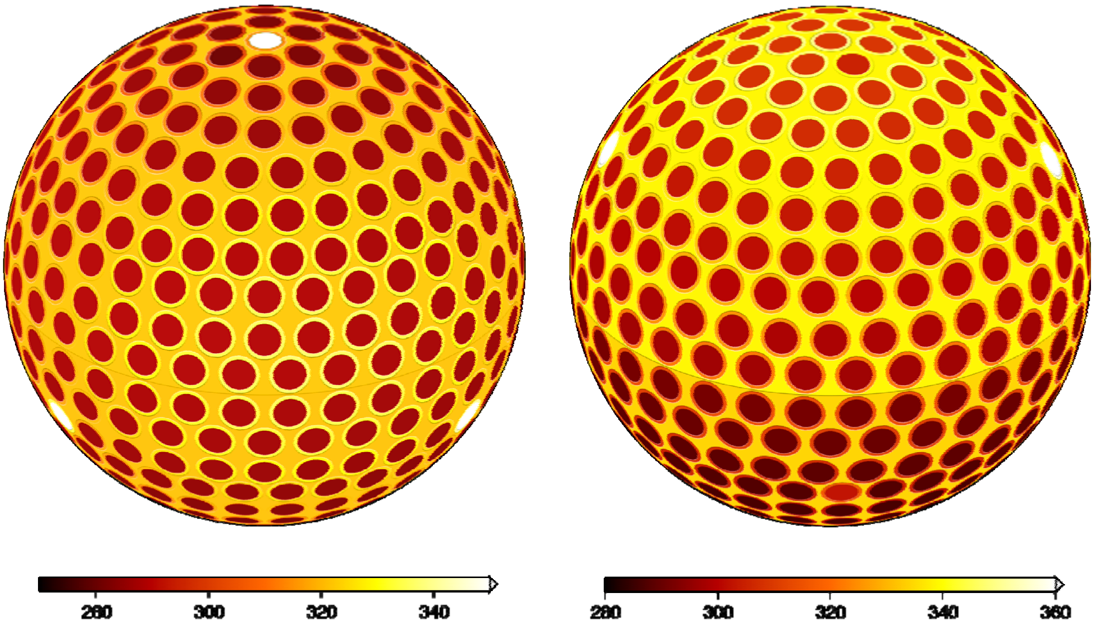

Figure 4. *Temperature distribution on January 1, 2002, for LAGEOS-1 and -2, respectively. All values are in Kelvin [Andrés et al., 2006].*

As an illustration, Figure 5 shows the long-term temperature behavior for a number of elements of LAGEOS-1; a similar behavior has been derived for LAGEOS-2 (not included here; cf. [Andrés *et al.*, 2006]). Figure 5(a) shows the temperatures for representative retainer rings and a Silicium CCR in the northern hemisphere. By virtue of its thermal properties, the CCR has an average temperature which is some 20 K lower than that of the retainer rings. All elements show a variation with time, which is correlated with the occurrence of solar eclipses (indicated by grey bands) and the position of the Sun (the main source of energy) w.r.t. the satellite spin axis; in the case of reflector assembly 89, which is located at a (satellite-fixed) co-latitude of about 58°, temperature variations are relatively humble, but after about 10 years in orbit the attitude of the spacecraft starts to develop into an erratic behavior w.r.t. λ and the spin rate drops off, resulting in extreme temperature variations for the retainer ring located at the satellite's north pole. A similar observation can be made for the retainer rings and the reflectors located in the southern hemisphere of the satellite (Figure 5(b)): the CCRs are typically cooler, show less variation, and big excursions of up to 60 K are visible for the retainer rings closer to the pole (in this case the south pole of the satellite). Figure 5(c), finally, very clearly illustrates the sensitivity of the Germanium CCRs to the actual lighting conditions: the 3 Ge CCRs that are located at co-latitude 121°, show a temperature variation of about 50 K (already large when compared to the behavior of the Si CCRs, cf. Figure 5(a)), but the situation appears to change dramatically for the CCR located at the very north pole of LAGEOS-1: temperature variations of up to 300 K are observed here.

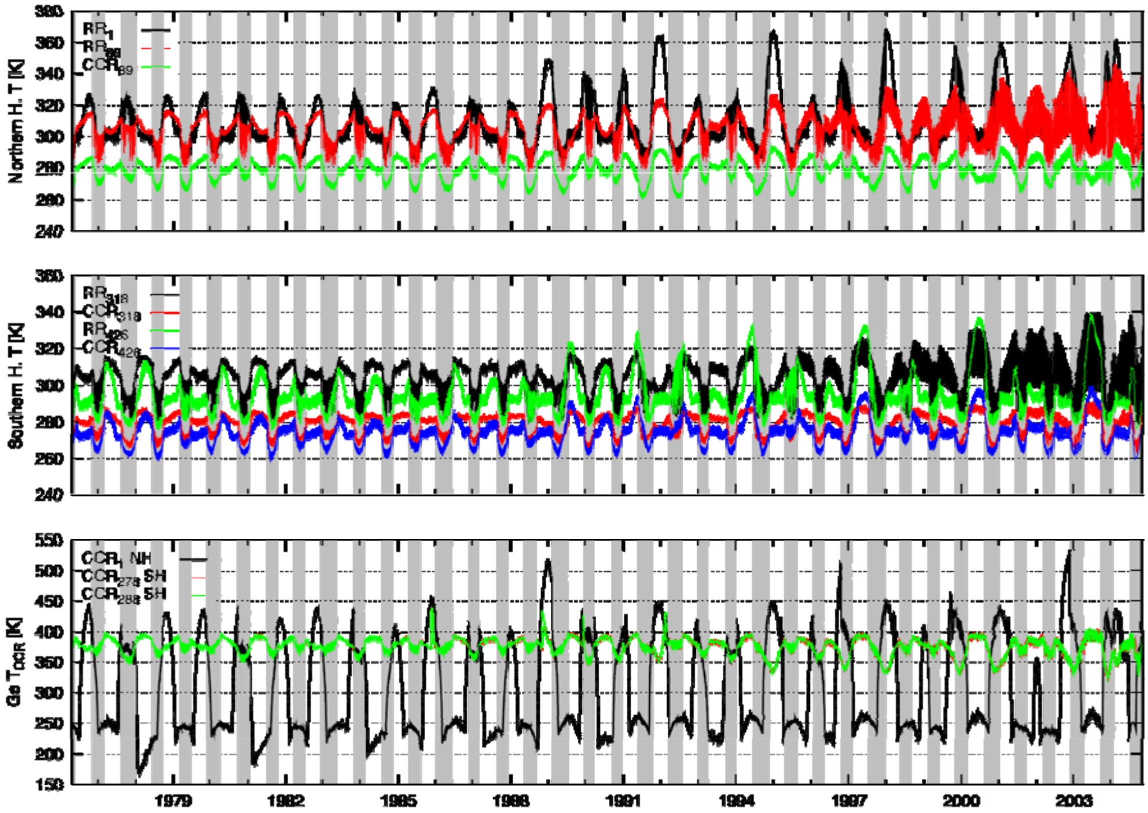

Figure 5. *Temperature behavior of several retainer rings and CCRs for LAGEOS-1 since launch [Andrés et al., 2006].*

Accelerations

Having arrived at a time-series of temperatures for the 2133 elements of each LAGEOS finite-element model, it is possible to derive values for the force that each element exerts (cf. [Slabinski, 1997]):

$$d\vec{F}_i \quad = \quad -\frac{2}{3}\,\frac{\varepsilon_i\,\sigma\,T_i^4}{c}\,dA_i\;\vec{n}_i$$

Integration of all contributions from all surface elements (clearly, internal elements do not contribute) yields the net thermal acceleration that each satellite experiences. An illustration of that is given in Figure 6: accelerations in the radial, along-track and cross-track directions for one day for LAGEOS-1 and LAGEOS-2, respectively; the right-hand side of the plots zooms in for a particular orbit during that day. It is clearly visible that for both satellites, radial and along-track accelerations of up to 50 pm/s^2 can be obtained (the two follow one another by virtue of the rotation of the orbital, satellite-related reference frame); much larger than the average value of about -3.4 pm/s^2 that is seen in the empirical (constant) accelerations. Since the cross-track orientation of the orbit remains more-or-less constant during one day, this component shows much less of a variation (but can have a very significant value). The plots indicate that an irregular behavior occurs in particular during times of eclipse; in such a situation, the cause for an uneven heating of the satellite disappears (ignoring any influence form the Earth, that is) and the net acceleration tends to develop towards zero.

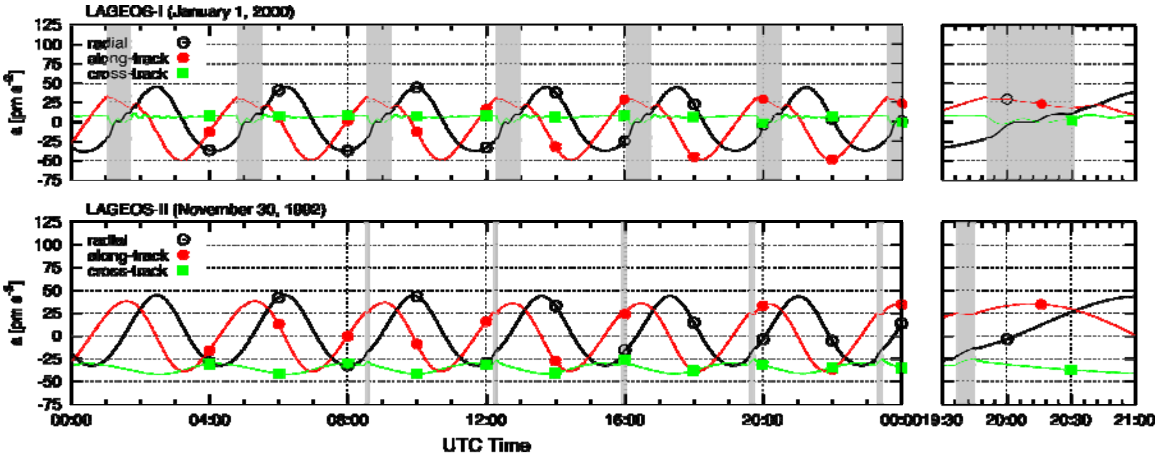

Figure 6. *Net thermal accelerations for (top) LAGEOS-1 on January 1, 2000, and (bottom) LAGEOS-2 on November 30, 1992. The grey bands indicate the exact periods when the satellites are in eclipse [Andrés et al., 2006].*

Extending the presentation to the full lifetime of the satellites (so far), Figure 7 shows the development of the net accelerations as well as the orientation of the Sun in a satellite frame, for each LAGEOS version. Again, the grey bands indicate when eclipses occur (somewhere in the orbit). Starting the discussion with LAGEOS-2 (Figure 7(b)), the long-term behavior is in line with what was shown in Figure 6 already: radial and along-track components interchange by virtue of the definition of the orbital frame, and the variation of the cross-track component is slower. All LAGEOS-2 components have values that go up to about 50 pm/s^2. In the situation that the Sun is located in the equatorial; plane of the satellite (i.e. $\beta_{\text{Sun-SA}}$ is equal to 90°), all 3 components of the net acceleration are effectively zero (by virtue of the rapid rotation of LAGEOS-2). As for LAGEOS-1, a similar story holds (Figure 7(a)), albeit that the relations are a bit more difficult to observe because of the longer time-span covered since launch. Also visible are the larger values for the net accelerations after about 1990, which is due to the specific rotational behavior of the spacecraft (with consequences for the temperature of particular elements of the satellite; cf. Figure 5). Although not included here explicitly, it can be shown that the model for the rotational behavior of the satellites plays a crucial role: net accelerations computed with the LOSSAM model (which is regarded as the state-of-the-art representation of the actual rotational behavior) differ by an amount of about 25 pm/s^2 with the results that would have been obtained with a more traditional (*i.e.* constant) model for the spin axis [Andrés *et al*., 2006].

Orbit computations

As a very first test of the actual usefulness of the results, two types of orbital computations have been done for LAGEOS-2 only (the choice of this satellite is arbitrary). First, weekly orbital fits have been computed using a model that does not include any external acceleration, and in which the solar radiation pressure force scaling parameter C_R is estimated only (in addition to the state-vector at epoch). Second, similar computations have been done but now with inclusion of the thermal accelerations as derived by the procedures sketched above (and keeping them fixed at their nominal values). Computations were done for the period October 1993 until December 1994. It should be emphasized here that no effort was done to fine-tune these results, nor to include other (necessary) elements to represent the orbital dynamics of the spacecraft. This explains the relatively high values for the rms-of-fit, which is shown in Figure 8 (typically, one would obtain fits in the order of better than 30 mm (for this period, that is), at the expense of solving for a collection of empirical accelerations; this was explicitly not the purpose of the current test). Figure 8 shows that the use of the thermal accelerations does lead to significant reductions in the quality of the orbit: the fit reduces from a range of 2.5-7.5 cm to a range of 2-4 cm, whereas the stability of the radiation scaling parameter C_R (a physical parameter, which should be constant rather than time-dependent – ignoring adjustments to the space environment during the first months in orbit [Ries *et al.*, 1997] indeed improves as well. The results shown here are very first results; further fine-tune of the computational model will hopefully result in the situation where the (estimation of) empirical accelerations can be discontinued altogether, without any loss of quality of the orbital solution nor of the derived parameters (origin, scale, station coordinates and such); preferably even an improvement of the latter products can be obtained.

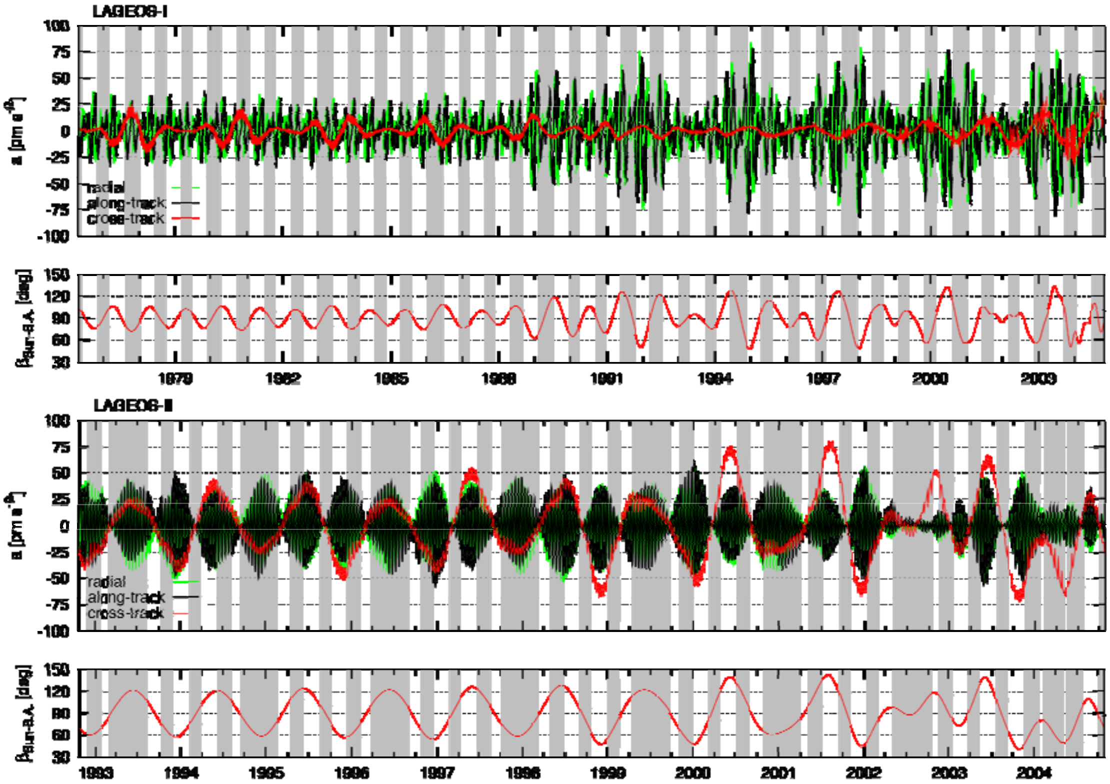

Figure 7. *Net thermal accelerations and solar co-latitude (i.e. position w.r.t. the satellite north pole) for (top) LAGEOS-1, and (bottom) LAGEOS-2. The grey bands indicate the exact periods when the satellites are in eclipse [Andrés et al., 2006].*

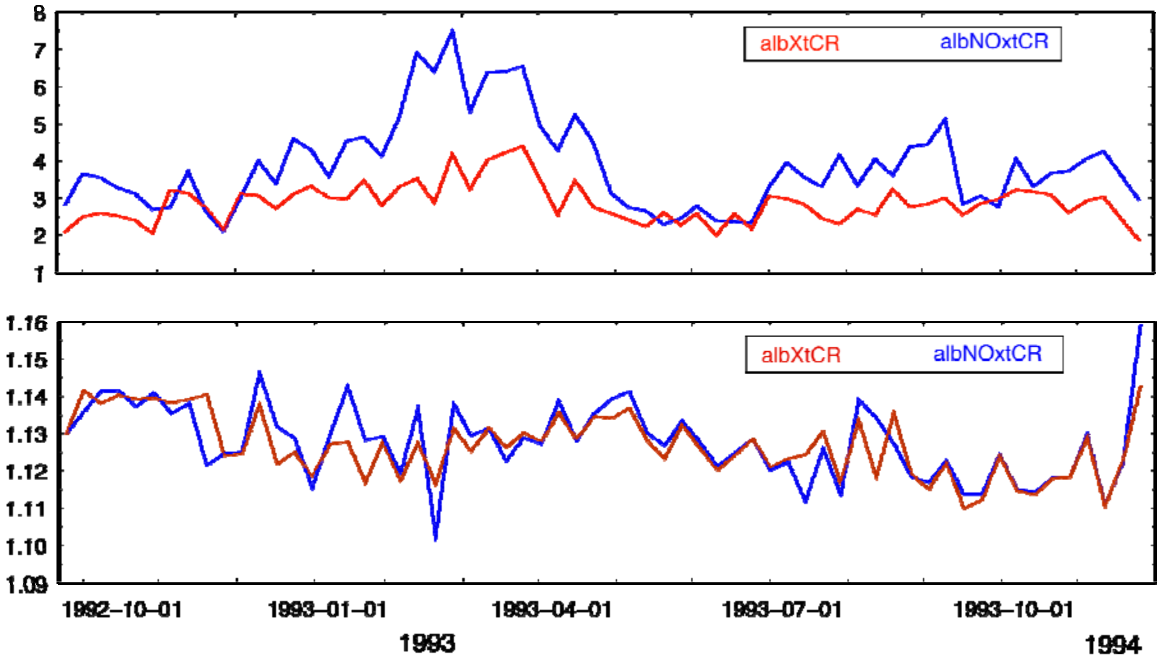

Figure 8. *Rms-of-fit (in cm) and solutions for the solar radiation scaling parameter C_R as a function of time for LAGEOS-2, with and without inclusion of the nominal thermal accelerations as shown in Figure 7.*

Conclusions and recommendations

Based on a detailed finite-element representation of the pair of LAGEOS satellites, and in combination with LOSSAM, the state-of-the-art model for the rotational behavior of each satellite, it has been possible to derive a highly accurate and unprecedented model for the thermal behavior of 2133 different components of each satellite: LOSTHERM. The temperatures appear to show a strong correlation with geometry w.r.t. the Sun as the main source of influx of energy. Also, temperature variations of up to several hundreds of Kelvin are observed by virtue of the sensitivity of particular spacecraft components to irradiation (absorption and emission coefficients). The instantaneous temperature distribution of the outer components in particular can be integrated to yield the net thermal acceleration. These accelerations have magnitudes of up to 75 pm/s^2, much larger than the average value that is typically obtained from orbital computations. The results clearly shows that the rotational behavior of the satellites plays a decisive role in the actual values of these accelerations, and underlines the neccessity of including such formulations in the most demanding orbital computations. It also underpins the need for continuation of independent observations of the rotational behavior of LAGEOS-2, and an answer to the challenge of doing similar things for LAGEOS-1.

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